Critical Speed and Voltage-Current Characteristics in Self-Field Plasma Thrusters

P. J. Turchi*
RDA Washington Research Laboratory, Alexandria, Virginia

To examine possible exhaust velocity limitations in high power magnetoplasmadynamic (MPD) arcjets, a power balance model is constructed. A special feature of this model is that the discharge thickness is inversely proportional to the velocity of the high-speed electrically conducting flow created by the discharge and its self magnetic field. Resistive dissipation then scales with electromagnetic thrust and flow speed. Ablation of arcjet materials adds to the total mass flow when the input propellant flow is insufficient to carry away the dissipated power. At high power levels, the exhaust velocity may plateau because of such mass addition at values comparable to Alfven critical speed. At lower powers, incomplete ionization and coupling of the injected neutral flow in the discharge can also result in such a velocity plateau (as interpreted by voltage and plume measurements). The voltage-current characteristics of MPD arcjets are discussed and research directions are indicated.

Nomenclature

B = magnetic (induction) field
E = first ionization level of propellant atom
g = geometric factor relating discharge volume to thruster area
j = current density
J = total current
J_{crit} = total current for which $\beta = 1$
m = mass of propellant atom
$\dot{m}$ = total mass flow rate
$\dot{m}_A/\dot{m}_F$ = mass flow rate of ablated material and propellant material, respectively
Q_A = heat absorbed per unit mass of ablated material
Q_F = heat absorbed per unit mass of propellant material
r = characteristic radius of thrust chamber
r_2/r_1 = ratio of outer-to-inner current attachment radii
R_M = magnetic Reynolds number
u = exhaust speed
u_{crit} = Alfven critical speed
u_0 = characteristic speed (for $\beta = 1$)
u_A = exhaust speed of ablation thruster ($\dot{m}_F \equiv 0$)
u_C = speed of electrically conducting portion of flow
$\bar{u}_e$ = electron flow velocity
u_I = ideal exhaust speed (full coupling, no ablation)
V = voltage
ρ = mass density
σ = electrical conductivity
μ = magnetic permeability
δ = characteristic discharge thickness
δ_A = characteristic discharge thickness for ablation arc
v = characteristic volume of discharge
Ω = ratio of ablated mass flow rate to input propellant mass flow rate
α = ablation parameter indicating relative ease of ablating material, $= Q_A/\epsilon Q_F$ (high α means ablation is difficult)
β = operating parameter indicating ideal thrust power relative to convective capability of input mass flow, $= (u_I/u_o)^2$ (high β means significant excess thermal power)
ϵ = fraction of excess thermal power (dissipated power minus power convected by propellant flow) causing ablation

Introduction

FOR several years, it has been suggested that the exhaust velocity of a magnetoplasmadynamic (MPD) arcjet is limited by the so-called Alfven critical speed[1]:

$$u_{crit} = (2E/m)^{1/2} \tag{1}$$

where E is the first ionization level of a fuel atom of mass m. A few theories, based on quite different physical models,[2-4] have led to this speed as a characteristic value of the exhaust velocity at which electrode erosion, insulator ablation, and/or "plasma instabilities" develop that serve to limit arcjet performance. In particular, for arcjet current J and mass flow $\dot{m}$, the "onset" of difficulties appears to occur at values of $J^2/\dot{m}$ that result in exhaust speeds near Alfven critical speed. The present discussion invokes the electromagnetic structure of the discharge flow and a requirement for power balance in steady state in order to derive self-consistent mass flow rates and exhaust speeds. Implications for voltage-current characteristics are described and further research directions are suggested.

Model

The basic equation for the current distribution in a longitudinal plasma flow is

$$\frac{D_e}{Dt}\left(\frac{B}{\rho}\right) - \frac{\nabla^2 B}{\rho\sigma\mu} \tag{2}$$

where ρ is mass density, σ electrical conductivity (assumed constant), and convective derivative is based on the electron fluid velocity $\bar{u}_e$. (For high-density flows, the plasma flow velocity $\bar{u}$ may be substituted for $\bar{u}_e$.) By dimensional analysis, the characteristic scale over which the magnetic field varies through the plasma flow in steady state is

$$\delta = 1/\sigma\mu u \tag{3}$$

Received Oct. 11, 1985; revision received April 5, 1986.

*Director.

Without flow, the magnetic field would decrease linearly between the upstream and downstream boundaries of the thruster. As the conducting flow accelerates, it convects magnetic flux away from the upstream region and concentrates flux near the downstream ($B=0$) boundary. A bifurcated current density distribution is thereby obtained. Higher flow speeds result in greater current concentration, and more intense dissipation. The consequence of such dissipation is increased ablation of electrodes and insulators. Ablation, however, contributes to the total mass flow rate and thereby lowers the flow speed, reducing the current concentration. A self-consistent solution should therefore be expected.

As a simple model, let convection provide the primary mechanism balancing resistive dissipation, with ablation absorbing the heat not carried away by the fuel mass flow

$$\dot{m}_A Q_A = \epsilon[(j^2 \nu/\sigma) - \dot{m}_F Q_F] \tag{4}$$

where j is the current density, ν the volume in which dissipation occurs, $\dot{m}$ the mass flow rate, and Q the heat absorbed per unit mass. Subscripts A and F refer to ablated and fuel mass, respectively. The factor ϵ allows energy to be lost without causing ablation (e.g., radiation to free space, thermal conduction).

The current density may be written in terms of the total current J and the characteristic scale δ, which also provides the volume, so

$$(j^2\nu/\sigma) = (1/\sigma)(J/2\pi r\delta)^2 \pi r^2 \delta g = (\mu/4\pi)J^2 ug \tag{5}$$

where g is a geometric factor that depends on the particular thruster, and u the exhaust speed. The electromagnetic thrust provides this speed:

$$u = (\mu/4\pi)(J^2/\dot{m})[\ln(r_1/r_2) + 3/4] \tag{6}$$

where $\dot{m}$ is the total mass flow rate $\dot{m}_F + \dot{m}_A$. Substitution in the equation for ablation results in a quadratic equation:

$$(\dot{m}_F + \dot{m}_A)[\dot{m}_F Q_F + \dot{m}_A (Q_A/\epsilon)]$$

$$= (\mu/4\pi)^2 J^4 g[(\ln(r_2/r_1) + 3/4)] \tag{7}$$

The ablation rate relative to the fuel flow rate is then

$$\Omega = (\dot{m}_A/\dot{m}_F) = \{[(\alpha+1)^2 + 4\alpha(\beta-1)]^{1/2} - (\alpha+1)\}/2\alpha \tag{8}$$

where $\alpha = Q_A/\epsilon Q_F$ and

$$\beta = [(\mu/4\pi)(J^2/\dot{m}_F)]^2 [g(\ln r_2/r_1 + 3/4)]/Q_F \tag{9}$$

For $\beta < 1$, there is no physical solution for the model as constructed. There is insufficient dissipation to account for the heat that could be carried away by the injected mass flow rate absorbing an energy per unit mass Q_F. This insufficiency can be resolved simply by reducing Q_F, i.e., allowing incomplete ionization of the injected flow. For $\beta > 1$, the fuel mass flow cannot cool the thruster sufficiently and ablation occurs. The degree of ablation and thus the exhaust velocity attained for a given current flow depends on the heat that can be absorbed by the ablated material, expressed relative to the fuel by the parameter α.

The condition $\beta = 1$ provides a value for $J^2/\dot{m}_F$ for which $\dot{m}_A = 0$ and the dissipation is exactly balanced by convection in the fuel mass flow. This value of $J^2/\dot{m}_F$ substituted in the equation for exhaust velocity gives

$$u = u_0 = [Q_F g(\ln r_2/r_1 + 3/4)]^{1/2} \tag{10}$$

for $Q_F \approx E/m$ and $g(\ln r_2/r_1 + 3/4) \approx 2$, $u_0 \cong u_{\text{crit}}$. Thus, correlation of self-field plasma thruster performance with Alfven critical speed might be expected simply on the basis of ablation when the fuel flow rate is insufficient for thruster cooling.

In Fig. 1, the actual speed (i.e., with ablation) is compared with u_I, the ideal exhaust speed (no ablation), as the ideal speed is increased relative to u_0. If there is no possibility of ablation ($\alpha = \infty$), then the exhaust speed can increase indefinitely. In the presence of plasma at 1-3 eV, however, solid surfaces can be expected to ablate and ionize about as readily as neutral fuel gas ($\alpha \approx 1$). Thus, a little excess heating is rapidly compensated by additional mass flow, resulting in an exhaust speed "plateau" at about $u_0 \approx u_{\text{crit}}$.

For $\alpha = 0$, which would correspond to an ablation-fed thruster, Fig. 1 indicates that the exhaust speed decreases relative to u_0 based on Q_F. Proper substitution of this limiting case ($\dot{m}_F = 0$) in the ablation cooling model, however, provides a constant operating speed

$$u_A = \{Q_A g[\ln(r_2/r_1) + 3/4]\}^{1/2} \tag{11}$$

that is basically equivalent to operation at $\beta = 1$. The characteristic discharge thickness for an ablation arc is then

$$\delta_A = (1/\sigma\mu u_A) \tag{12}$$

Incomplete Ionization and Coupling

As noted earlier, the situation of $\beta < 1$ can be associated with incomplete ionization of the input mass flow. If the mean free path for momentum exchange between ions and neutrals is small compared to the discharge and thruster dimensions and the input flow is well mixed into the discharge, then it would be appropriate to divide the electromagnetic thrust by the full mass input rate. The actual exhaust speed should then follow the ideal speed as indicated in Fig. 1 for $\beta < 1$. With poor mixing, however, or at low densities, the ions can slip relative to the neutrals, and the velocity of the electrically conducting fluid can be higher than the ideal speed. In the limit of complete slip, the mass flow subjected to the electromagnetic thrust scales with dissipation. The situation is completely analogous to the pure ablation-fed thruster, with electrical dissipation effectively "ablating" the input neutral mass flow. The conductor speed will be given by:

$$u_C = \{Q_F g[\ln(r_2/r_1) + 3/4]\}^{1/2} \tag{13}$$

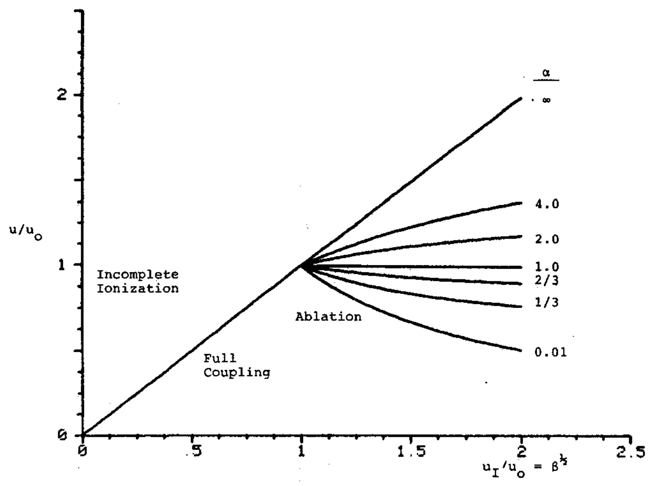

Fig. 1 Variation of actual speed with expected speed, both relative to critical speed u_0, as a function of energy per unit mass for ablation relative to fuel, $\alpha = Q_A/\epsilon Q_F$. (Labels refer to regimes of operation discussed in text.)

In the ion slip limit, the ionized fraction of the flow would achieve a high speed at modest current levels and would exhibit constant speed until $\beta = 1$ [which corresponds to complete (single) ionization of the input mass flow].

Voltage-Current Characteristics

To the extent that the voltage across an MPD arcjet largely reflects the electromotive ($\bar{u} \times B$) effects of plasma flow at significant magnetic Reynolds number, the arcjet voltage-current characteristic should display regimes of operation associated with the velocity of the conducting flow. In Fig. 2, the components of the arcjet voltage are sketched. Anode and cathode falls provide a significant bias that may be nearly constant with respect to total current (although interesting variations, particularly of the anode fall, might be expected). In idealized MPD operation, for which the mass flow is only the input mass flow (no ablation) and a single heavy-particle velocity exists (full ionization and/or no ion slip), the conducting flow speed scales as J^2, so the voltage $V \sim uB \sim J^3$. At high current levels ($\beta > 1$) and finite α, however, the mass flow includes ablated material and the speed increases less rapidly with current, perhaps even becoming limited to a constant value. The back EMF component of voltage will then scale linearly with current.

If the input mass flow is not fully ionized and collisionality between ions and neutrals is not sufficient to prevent significant ion slip, then the flow speed that determines the back electromotive force (EMF) will be higher than that computed from the electromagnetic thrust divided by the full input mass flow rate. Since this flow speed will also be constant, scaling with $Q_F^{1/2}$, the back EMF voltage will increase linearly with J at a level above that for the idealized J^3 variation. Intermediate degrees of collisionality between full slip and complete ion-neutral coupling provide voltage-current characteristics between the linear and cubic variations.

If variations in conductivity and current path length with total current level (and mass flow rate) are ignored compared to the basic variation of discharge thickness with arcjet operation, then two cases can be identified for the resistive component of the voltage. For high magnetic Reynolds number flow, the discharge thickness scales as $(\sigma\mu u)^{-1}$, so the resistance will increase as u and the resistive voltage will vary as uJ. This variation will therefore be included in the same manner as the back EMF.

In some regions of the flow, particularly near the propellant inlet, where R_M is low, the current distribution may be determined by processes such as ionization and thermal

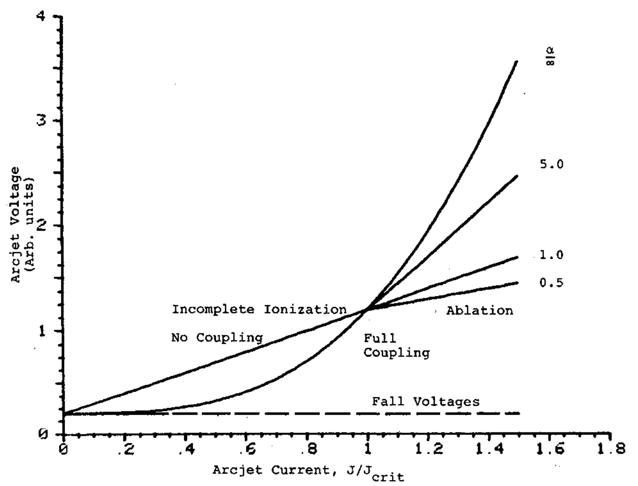

Fig. 2 Sketch of MPD arcjet voltage-current characteristics. Voltage V has arbitrary units and current J is relative to critical current J_{crit} for which $u_I = u_0$. $(J/J_{crit})^2 = u_I/u_0 = \beta^{1/2}$. (Labels refer to regimes of operation discussed in text.)

transport (i.e., conductivity variations in the direction of the flow are important). If the discharge thickness is constant, for example, then the resistive component of voltage will increase linearly with current and might combine with the back EMF voltage for constant flow speed (due either to incomplete ionization and coupling, or significant ablation).

The total voltage-current characteristic for an MPD arcjet can display a considerable variation depending on the particulars of geometry (e.g., inlet port location and shape) and material selection (α value, collision cross-sections). A linear increase of voltage with current, above constant fall voltages, above constant fall voltages, may be expected at low magnetic Reynolds number and for incomplete ionization and coupling. Transition to cubic variation of voltage with current should then occur as full ionization ($\beta = 1$) is attained, but may be achieved at lower values of current if the magnetic Reynolds number and degree of ion-neutral collisionality are both high. This transition, however, may be obscured if ablation becomes important and the voltage returns to a linear variation with current. Two quite distinct aspects of arcjet design (inlet geometry and insulator material) could thus combine to provide a rather linear voltage-current characteristic for the range of experimental operation.

To the extent that the arcjet current is maintained by the relatively high impendance of the power source, temporal variations in the back EMF component of the voltage reflect similar variations in flow speed. Such flow speed variation affects the current density distribution, i.e., the discharge thickness ($\delta \sim u^{-1}$). At high power levels, significant variations in ablation may then occur which in turn result in flow speed variations. As a simple example, suppose that the coupling of dissipation to ablatable surfaces (as expressed by the efficiency ϵ) changes abruptly from $\epsilon = 1$ for a high-speed flow (no ablation) to $\epsilon = 0$, if ablation lowers the flow-speed and thereby broadens the discharge. The arcjet flowfield could then flicker in times scaled by the chamber length divided by the flow speed. Voltage oscillations at a few hundred kilohertz would then be observed (with amplitudes corresponding to the voltage differences between different α values in Fig. 2). Such oscillation in arcjet voltage and flowfield could significantly increase the transport of heat and mass, further degrading performance.

Conclusions

The modeling based on power balance in magnetoplasmadynamic (MPD) arcjet operation is essentially a dimensional argument that indicates scaling relationships. To predict from first principles the detailed behavior of an arcjet of particular geometry, material choice, and propellant type would require a substantially more complex calculational tool, namely a two-dimensional computer code. The present state of the art for such codes is quite sufficient for hydrodynamic calculations including radiation transport, plasma chemistry, and interactions with solid boundaries. Collisionless motion of ensembles of charged particles can also be computed in self-consistent electric and magnetic fields. Estimates of collisional mean free paths vs arcjet dimensions,[5] however, indicate that at least portions of the MPD arcjet operate in a transition flow regime, possibly with nonequilibrium (i.e., evolving) chemistry. Phenomena such as ion slip and incomplete ionization and/or mixing of the injected neutral flow may be inaccessible to both hydrodynamic and collisionless codes.

In lieu of detailed calculations, the present modeling attempts to provide a structure to support understanding of experimental arcjet behavior. Experimental measurements of ion vs neutral velocities as a function of position within the arcjet chamber would be useful to assess the degree of collisionality (and incidentally the degree of ionization). Correlation of collisionality and ionization with overall arcjet behavior (e.g., relationship to onset of noise on voltage signals) and with variations in chamber geometry, especially the fuel injection arrangement, could then lead to improved criteria for MPD arcjet design.

References

[1]Alfven, H., "Collision Between a Nonionized Gas and a Magnetized Plasma," *Review of Modern Physics,* Vol. 32, 1960, p. 710.

[2]Malliaris, A. C., John, R. R., Garrison, R. L., and Libby, D. R., "Quasi-Steady MPD Propulsion at High Power," NASA CR-111872, (Avco AVSD-0146-77-RR), Feb. 1971.

[3]Cann, G. L and Harder, R. L., "Thrust Efficiencies of Electromagnetic Engines," *AIAA Journal,* Vol. 6, March 1968, p. 558.

[4]Devillers, P. and Burton, R. L., "The Importance of Alfven's Critical Velocity for MPD Arcs," AIAA Paper 70-1100, 1970.

[5]Turchi, P. J., "The Cathode Region of a Quasi-Steady Magnetoplasmadynamic Arcjet," Department of Aerospace and Mechanical Sciences, Princeton University, Princeton, NJ, Rept. AMS 94 OT, Sept. 1970, p. 42.